Argument Forms

Validity and Invalidity

We now take up the subject of arguments. Traditionally arguments have been divided into deductive and inductive. Several ways of formulating the distinction have been tried, none of which are satisfactory. In a deductive argument the conclusion is contained in the premises; in an inductive argument the conclusion goes beyond the premises. A deductive argument necessitates the conclusion; an inductive argument confers support to the conclusion. A deductive argument is explicative; an inductive argument is amplitive. For reasons more easily explained later, it is best to drop the deductive/inductive distinction altogether. Instead, we will divide all arguments into the valid and the invalid.

A valid argument is a substitution instance of a valid argument form. A valid argument form is one with no possible substitution instance having true premises and a false conclusion. An invalid argument form is one that does have substitution instances with true premises and a false conclusion.

Argument forms can be tested for validity by using truth tables. To do so, list each of the premises and the conclusion, and construct truth columns for each as below. Here we test the argument form,

$$\begin{array}{l} p \supset q \\ \underline{q \supset r} \\ \therefore p \supset r \end{array}$$

			premise	premise	conclusion
p	q	r	$p \supset q$	$q \supset r$	$p \supset r$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

The conclusion is false in lines 2 and 4. In each of these lines, there is also a false premise. Since there is no substitution instance with true premises and a false conclusion, the argument form is valid.

Compare the above argument form with the following form.

$$\begin{array}{l} p \supset q \\ \underline{\sim p} \\ \therefore \sim q \end{array}$$

		premise	premise	conclusion
p	q	$p \supset q$	~p	~q
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Line 1 has a false conclusion, but also a false premise. Line 3 is the significant substitution instance, for the conclusion is false and the premises are both true. This argument form is invalid.

A substitution instance of an argument form with true premises and a false conclusion is a counter example to the argument. Any argument for which there is a counter example is an invalid argument. Any argument for which there is no possible counter example is a valid argument.

Exercise 5.1
Use truth tables to test the validity of each of the following argument forms.

1. $p \vee q$
   $\underline{\sim p}$
   $\therefore q$

2. $p \supset q$
   $\underline{q \supset r}$
   $\therefore r \supset p$

3. $(p \,\&\, q) \supset r$
   $\underline{\sim r}$
   $\therefore \sim q$

4. $\underline{p}$
   $\therefore q \vee \sim q$

5. $\underline{p \,\&\, \sim p}$
   $\therefore q$

6. $p \supset q$
   $\underline{p \,\&\, r}$
   $\therefore q \,\&\, r$

7. $\underline{p \,\&\, (q \vee r)}$
   $\therefore (p \vee q) \,\&\, (p \vee r)$

8. $\underline{p \vee (q \vee r)}$
   $\therefore p \vee r$

9. $\underline{p \,\&\, (q \,\&\, r)}$
   $\therefore p \,\&\, r$

10. $\sim p \vee q$
   $\underline{\sim q \vee r}$
   $\therefore \sim p \vee r$

Short Truth Table Method

When dealing with an invalid argument form, it is not necessary to construct the entire truth table to prove invalidity. We need only show that there is one substitution instance where the premises are true but the conclusion is false. Let's use as an example the argument form,

$$\underline{p \lor (q \lor r)}$$
$$\therefore\ p \lor r$$

To demonstrate the invalidity of this argument form, we need one set of values for the variables, *p*, *q*, and *r*, such that $p \lor r$ is false but $p\lor (q \lor r)$ is true. If we assign the value *false* to *p* and *r*, then the conclusion is false. If we then assign the value *true* to *q*, then the premise, $p \lor + (q \lor r)$ is true. Compare this result to *#8* in the previous exercise.

To use the short truth table method, it is easiest to write down the argument and a place to assign values beside it, such as below.

$$\begin{array}{ll} p \supset q & \\ q \supset r & \underline{p\ \ q\ \ r} \\ \underline{\sim q\quad} & \\ \therefore\ \sim r & \end{array}$$

Usually it will be easiest to begin by making the conclusion false. So assign *true* to *r*. In order for the third premise to the true, *false* must be assigned to *q*. Since *q* is already assigned *false* and *r* assigned *true*, the second premise is true. Finally, since *q* is assigned *false*, *p* must be assigned *false* for the first premise to be true. The counter-example for the above argument is

p	q	r
f	f	t

Exercise 5.2

Use the short truth table method to demonstrate the invalidity of the following argument forms.

1. $$\begin{array}{l} p \equiv \sim q \\ p \supset r \\ \underline{r\qquad} \\ \therefore\ \sim q \end{array}$$

2. $$\begin{array}{l} p \supset q \\ s \supset q \\ \underline{r \supset s\quad} \\ \therefore\ r \supset p \end{array}$$

3. $r \;\underset{\cdot}{\vee}\; s$
$s \vee t$
$\underline{p \vee q \quad}$
$\therefore\ p \vee t$

4. $\sim p \supset q$
$\underline{\sim q \supset r \quad}$
$\therefore\ \sim p \supset \sim r$

5. $(p \,\&\, q) \vee r$
$\underline{p \,\&\, r \qquad}$
$\therefore\ q$

6. $(p \,\&\, q) \supset r$
$\sim q$
$\underline{\sim r \qquad\qquad}$
$\therefore\ \sim p$

7. $p \supset (q \supset r)$
$\underline{\sim q \qquad\qquad}$
$\therefore\ \sim p \supset \sim r$

8. $p \supset (q \,\&\, r)$
$r \supset s$
$\underline{s \supset q \qquad}$
$\therefore\ s \supset \sim p$

9. $p \equiv (q \vee r)$
$\sim(q \,\&\, s)$
$\underline{\sim r \qquad\qquad}$
$\therefore\ \sim p$

10. p
$\underline{q \qquad\qquad\qquad\qquad}$
$\therefore\ r \vee [s \vee (t \vee u)]$

Validity and Logical Implication

Another way to define validity is in terms of logical implication. The premises of a valid argument logically imply the conclusion. This relation of logical implication between premises and conclusion can be demonstrated by truth tables. To do so, we conjoin the premises of an argument, making that the antecedent of a condition statement of which the conclusion is the consequent. If the argument is valid, the resulting conditional will be a tautology.

Given the argument form,

$p \supset q$
$\underline{p \qquad}$
$\therefore\ q$

we construct the conditional statement, $[(p \supset q) \,\&\, p] \supset q$. We then test the conditional with the truth table,

p	q	$[(p \supset q)$	$\& \; p]$	$\supset q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Since the resulting conditional is a tautology, the argument is valid. There is no possible substitution instance for the statement form where the antecedent, the conjunction of the premises, is true and the consequent, the conclusion, is false. The premises logically imply the conclusion.