Multiple Generality

Any statement, which contains more than one quantifier, is a multiply general statement. Arguments consisting of multiply general statements can be handled with the reduction tree procedures we have already developed. Extra care, however, must be exercised in instantiating multiply general statements.

Let's begin with the argument, "Everybody loves somebody. Therefore John loves somebody." We will use the notation, Lxy: x loves y; j: John. We get the following tree.

	1.	∀x ∃y Lxy	premise
√	2.	~∃x Ljx	~ conclusion
	3.	∀x ~Ljx	2, QN
√	4.	∃y Ljy	1, UI
	5.	Lja	4, EI
	6.	~Lja	3, UI
		x	

By instantiating line 1, in line 4, to j, we are able to use the instantiation of line 3, in line 6, to close the path. We might have first instantiated line 3 to j, giving us ~Ljj, but it would have been a useless step.

In the above treatment of the argument, the universe of discourse was assumed to be persons. If we make personhood explicit by adding to the notation Px: x is a person, we might have, mistakenly, analyzed the argument as in the tree below.

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      √ 6.  ~(Pj & .
          /    \
7.  ~Pj      ~Ljj   6, denie.
     o         o
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As we see, the argument thus interpreted is invalid. In the open paths, Pj is false. If John is not a person (~Pj), then the set consisting of premises and denial of the conclusion is satisfiable! Of course, John is a person (Pj), and we can add that to the premises to make the argument valid.

An argument with one or more unstated premises is called an enthymeme. Any invalid argument can be treated as an enthymeme and made valid by the addition of premises. Trivially, this can be done by adding the conclusion to the list of premises. Ideally, we want to identify those tacit presuppositions whose identification is necessary to make arguments explicit.

Exercise 16.1

Symbolize the following arguments using the suggested notation. Identify any missing premises, and if necessary expand the notation. Then construct reduction trees of the argument as expanded with the addition of premises.

1. Bill is the twin of Will. Therefore Will is the twin of Bill. (b: Bill; w: Will; Txy: x is the twin of y)
2. Mary is the sister of John. John is the brother of Harry. Therefore Mary is the sister of Harry. (m: Mary; j: John; h: Harry; Bxy: x is the brother of y; Sxy: x is the sister of y)
3. Jonesville is north of Campton. Hollander is south of Campton. Therefore Hollander is south of Jonesville. (j: Jonesville; c: Campton; h: Hollander; Nxy: x is north of y; Sxy: s is south of y)

4. Kansas is more populous than New Mexico. Pennsylvania is more populous than Kansas. Therefore Pennsylvania is more populous than New Mexico. (k: Kansas; m: New Mexico; p: Pennsylvania; Pxy: x is more populous than y)
5. Bob is the father of Mary and George. Therefore Mary is George's sister. (b: Bob; m: Mary; g: George; Fxy: x is the father of y; Sxy: x is the sister of y)