Polyadic Predication

In the previous chapter, the predicates we dealt with were all one place or monadic. The statement,

Socrates is wise.

attributes the predicate wise to the subject Socrates. "Ws" is sufficient to symbolize the statement. A more complicated statement to analyze is

Socrates met Alcibiades.

"Having met" is a relation one individual has to another. It is a two place or dyadic predicate. Using the notation, s: Socrates, a: Alcibiades, and Mxy: x met y, we would symbolize the statement as

Msa

Logical complexity can hide beneath grammatical simplicity. Consider the statement

John and Mary are friends.

On the surface, it might appear that it could be rendered

Fj & Fm

where Fx: x is a friend, j: John, and m: Mary. The problem is that one cannot be a friend without being a friend of someone. Friendship is dyadic. We need a predicate such as Fxy: x is a friend of y. The expression

Fjm

is still not sufficient, however. It says that John is a friend of Mary, but our original sentence expresses reciprocity in the relation. Thus we need

Fjm & Fmj

Polyadic predicates can occur in general statements as below, where Lxy: x loves y, Px: x is a person, and j: John.

John loves somebody.	$\exists x\,(Px \,\&\, Ljx)$
Somebody loves everybody.	$\exists x\,[Px \,\&\, \forall y\,(Py \supset Lxy)]$
Everybody loves somebody.	$\forall x\,[Px \supset \exists y\,(Py \,\&\, Lxy)]$
John loves everybody but himself.	$\forall x[(Px \,\&\, x{\neq}j) \supset Ljx]$

We are by no means limited to one or two place predicates. Sentences with both direct and indirect objects express three place predicates. Consider the example

Mary gave John a rose.

The predicates in this case are Rx: x is a rose, and Gxyz: x gave z to y. This would be symbolized

$$\exists x\,(Rx \,\&\, Gmxj)$$

Exercise 15.1

Symbolize the following using the suggested notation.

j: John
f: Fido
Dx: x is a dog
Px: x is a pointer
Qx: x is a quail
Rx: x is a retriever

Bxy: x belongs to y
Fxy: x finds y
Sxy: x shoots y
Cxyz: x carries y to z

1. All John's dogs are retrievers.
2. All John's dogs are either retrievers or pointers.
3. Some of John's dogs are retrievers and some are pointers.
4. Some of John's dogs are retrievers and the rest are pointers.
5. If a pointer of John's finds a quail, John will shoot it.
6. If John shoots a quail, then a retriever of John's will carry it to him.
7. Fido is John's retriever, not pointer.
8. If a dog of John's carries a quail to him, then the dog is Fido.
9. John has a dog other than Fido, which is a pointer.
10. Fido is John's only dog that is not a pointer.

Properties of Relations

The analysis of statements and arguments involving polyadic predicates or relations often depends upon identifying certain properties of those relations. If Jack is the twin of Bill (Tjb), then Bill is the twin of Jack (Tbj). The relation, *is the twin of*, is a symmetrical relation. Any relation, *R*, is symmetrical if and only if:

$$\forall x \forall y\ (Rxy \supset Ryx)$$

If Jack is the father of Bill (Fjb), then Bill cannot be the father of Jack (~Fbj). The relation, *is the father of*, is an asymmetrical relation. Any relation, *R*, is asymmetrical if and only if:

$$\forall x \forall y\ (Rxy \supset {\sim}Ryx)$$

If a relation is neither symmetrical nor asymmetrical, then it is nonsymmetrical.

If Jack is older than Bill (Ojb), and Bill is older than Harry (Obh), then Jack is older than Harry (Ojh). The relation, *is older than*, is a transitive relation. Any relation, *R*, is transitive if and only if:

$$\forall x \forall y \forall z\ [(Rxy\ \&\ Ryz) \supset Rxz]$$

If Jack is the father of Bill (Fjb), and Bill is the father of Harry (Fbh), then Jack is not the father of Harry (~Fjh). The relation, *is the father of*, is an intransitive relation. Any relation, *R*, is intransitive if and only if:

$$\forall x \forall y \forall z\ [(Rxy\ \&\ Ryz) \supset {\sim}Rxz]$$

If a relation is neither transitive nor intransitive, it is nontransitive.

If Jack is the same age as Bill (Ajb), it follows that Jack is the same age as himself (Ajj). The relation, *the same age as*, is a reflexive relation. Any relation, *R*, is reflexive if and only if:

$$\forall x\ [\exists y\ (Rxy \vee Ryx) \supset Rxx]$$

If Jack is the brother of Bill (Bjb), if follows that he is not the brother of himself (~Bjj). The relation, *is the brother of*, is an irreflexive relation, a relation nothing has to itself. Any relation, *R*, is irreflexive if and only if:

$$\forall x\ {\sim}Rxx$$

A relation is totally reflexive if and only if everything has that relation to itself; that is, a relation, *R*, is totally reflexive if and only if

$$\forall x\ Rxx$$

Identity is a totally reflexive relation. If Jack is identical to Bill ($j = b$), then Jack is identical to himself ($j = j$). If a relation is neither reflexive nor irreflexive, it is nonreflexive.

Exercise 15.2

Identify the each of the properties of the following relations.

1. is greater than
2. is less than
3. is equal to
4. is ahead of
5. is beside
6. is inside of
7. is beneath
8. is lighter than
9. evolved from
10. is married to
11. is a higher rank than
12. is a sibling of
13. is an uncle of
14. is a sister of
15. is secretary to
16. is employed by
17. taught
18. played against
19. negotiated with
20. aided