Immediate Inferences

Equivalence

We will now examine two relationships that may hold between statements. These relations are called immediate inferences since they can be used to construct arguments having only one premise. The first of these is logical equivalence. To explain logical equivalence, we will have to introduce some other concepts first. A statement is defined as expressible by a sentence and either true or false. Since a statement is either true or false, we can say that every statement has a truth value. The truth value of a true statement is *true*. The truth value of a false statement is *false*.

A preliminary definition of logical equivalence is that two or more statements are logically equivalent if and only if they necessarily have the same truth value. We can intuitively understand what it is for two statements to accidentally have the same truth value. The statements

Ronald Regan was elected president of the United States.
Howard Baker served as Republican leader of the Senate.

both happen to be true. However, if James Carter had won re-election, the first might have been false. If Howard Baker, in his youth, had decided to be a college professor, the second might have been false. The statements,

William Shakespeare was a British philosopher.
Mark Twain authored a play about the prince of Denmark.

both happen to be false. Nevertheless, we can easily imagine circumstances under which they might have been true. What it means for two statements to necessarily have the same truth value is not so obvious.

The logical relation in virtue of which two or more statements necessarily have the same truth value is a function of the forms of the statements. In the previous chapter, you learned to identify the four forms of categorical statements. Thus you recognize that the statements,

All humans are rational animals.
All birds are feathered animals.

have the same form. They are both A form categorical statements. Another way of saying they have the same form is to say that they are substitution instances of the same statement form.

Two statements are substitution instances of the same form if they each result from replacing the variables in the form with constants. The form or schema for an A categorical statement is

All S are P.

If the "S," which is a variable for the subject term, is replaced with "humans," and the "P," which is a variable for the predicate term, is replaced with "rational animals," the result is

All humans are rational animals.

If the "S" is replaced with "birds," and the "P" is replaced with "feathered animals," the result is

All birds are feathered animals.

The two statements are substitution instances of the same statement form. A statement form or schema can be thought of as a blueprint or pattern for constructing statements.

Two or more statements are said to be *simultaneous* substitution instances if they are the result of substituting the same constants for the same variables in two or more statement forms. Thus, given the two statement forms,

All S are P
Some S are P.

and substituting "lions" for each "S" and "felines" for each "P," we get the simultaneous substitution instances,

All lions are felines.
Some lions are felines.

Now we can offer a better definition of logical equivalence. Two statements are logically equivalent if and only if they are simultaneous substitution instances of logically equivalent statement forms.

Obversion

There are operations which we can perform upon categorical statements or statement forms which yield logical equivalences. In order to describe obversion, we need the concepts of the universe of discourse and the complement of a class. The universe of discourse is entire range of things about which the discussion is taking place. The complement of a class is everything in the universe of discourse that is not a member of that class. The complement of the class of humans is everything that is not human. We denote the complement of a class with the prefix "non-." Thus the complement of the class of humans is the class of non-humans.

Complementarity is a symmetrical relation. That is, if some class A is the complement of B, then B is also the complement of A. For this reason, we would not have to denote the complement class of non-humans as the class of non-non-humans. It is simply the class of humans.

Forming the obverse of a categorical statement is a two-step operation. The first step is to change the quality of the statement. If a statement is affirmative, its obverse is negative. If a statement is negative, its obverse is affirmative. The second step is to replace the predicate term with its complement. The symbol "⇔" is read as "is logically equivalent to." The following table shows the four forms of categorical statements with their obverses.

		Obverse
All S are P.	⇔	No S are non-P.
No S are P.	⇔	All S are non-P.
Some S are P.	⇔	Some S are not non-P.
Some S are not P.	⇔	Some S are non-P.

The use of the prefix, "non-," while logically precise, is not elegant. Wherever possible, use ordinary English expressions for the complement of a class. If the universe of discourse is people, then the complement of "men" should be expressed as "women," not non-men. The complement of "adults" is "children."
Let's use as an example the statement,

All students who received A's are students on the Dean's list.

The universe of discourse is students. This statement says nothing about faculty, alumni, or office furniture. The predicate class is *students on the Dean's list*. The complement of that class is *students not on the Dean's list*.
The obverse of the statement is

No students who received A's are students not on the Dean's list.

Exercise 10.1

Write of obverse for each of the following categorical statements.

1. All voters are citizens.
2. All players are people permitted on the field.
3. Some seniors are non-graduates.
4. No alumni are non-graduates.
5. Some seniors are not eligible players.

Conversion

The converse of a categorical statement is formed by reversing the subject and predicate terms. The converse of "No people are unicorns" is "No unicorns are people." The converse of "Some people are unicorns is "Some unicorns are people."

The converse is logically equivalent to the original statement only in the cases of the E and I forms. That the A and O forms are not equivalent to their converse is obvious from the following counter-examples.

All freshmen are students.
All students are freshmen.

Some runners are not winners of the race.
Some winners of the race are not runners.

The first sentence in each case expresses a true statement, while the second sentence expresses a false statement. Since the members of each pair have opposite truth values, they cannot be equivalent.

The following table shows the E and I forms with their converses.

Converse		
No S are P.	⇔	No P are S.
Some S are P.	⇔	Some P are S.

Exercise 10.2

Write of converse of the of the following if and only if the converse is equivalent to the original.

1. Some snakes are good pets.
2. Some snakes are not good pets.
3. No non-mammals are good pets.
4. All snakes are non-mammals.
5. No man-eating tigers are good pets.

Contraposition

Forming the contrapositive of a categorical statement is also a two step operation. The first step is to replace the subject term with the complement of the predicate. The next step is to replace the predicate term with the complement of the subject.

To form the contrapositive for

All humans are rational beings

we replace the subject term, *humans*, with *irrational beings*, and the predicate term, *rational beings*, with *non-humans* . This gives us the contrapositive, *All irrational beings are non-humans.*

To form the contrapositive of

Some students are not logicians

replace the subject term, *students*, with *non-logicians* , and the predicate term, *logicians* , with *non-students* . This gives the contrapositive,

Some non-logicians are not non-students.

The following table shows the A and the O forms with their contrapositives.

	Contrapositive	
All S are P.	⇔	All non-P are non-S.
Some S are not P.	⇔	Some nonP are not non-S.

The contrapositive is equivalent to the original statement only in the case of the A and O forms.

Exercise 10.3

Write the contrapositive of each of the following.

1. All Spartans are politicians.
2. Some Spartans are not politicians.
3. Some logicians are not mathematicians.
4. All botanists are biologists.

Write the obverse, converse, and contrapositive of each of the following if and only if the resulting statement is logically equivalent to the original.

1. All capitalists are democrats.
2. Some capitalists are democrats.
3. No capitalists are democrats.
4. Some capitalists are not democrats.

Implication

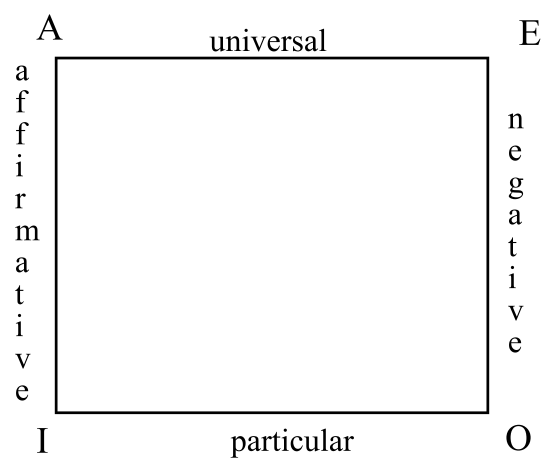
The next relation between statements which we will examine is logical implication. A statement *A* logically implies another statement *B* if and only if it is impossible for *A* to be true and *B* to be false. Implication is a weaker relation than equivalence. Logical equivalence requires that both statements necessarily have the same truth value. Implication requires both statements have the same truth value only in the case where the first is true.

Exercise 10.4

Logical equivalence can be described as mutual implication. If *A* and *B* are logically equivalent, then *A* logically implies *B* and *B* implies *A*. Prove this to be the case.

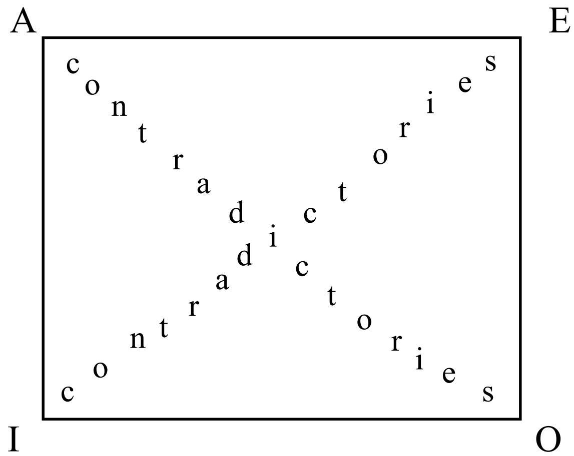
The Traditional Square of Opposition

The logical implications of traditional categorical logic are described graphically by the traditional square of opposition. The four forms are placed at the corners of a square as below.



Universal forms are at the top; particular forms are at the bottom. Affirmative forms are on the left; negative forms are on the right.

The first relation we will define on the traditional square is that of contradiction. Contradictories are such that they cannot have the same truth value. The relation of contradiction holds between the A and O, and the E and I.

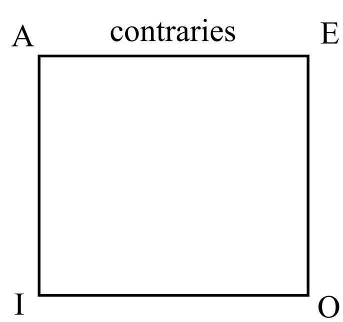


The truth of one contradictory implies the falsity of the other; the falsity of one implies the truth of the other.

Exercise 10.5

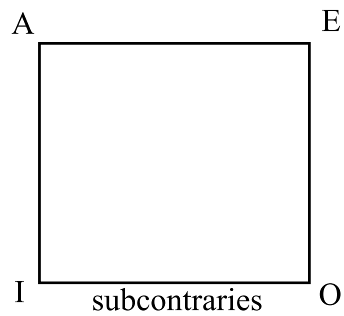
Any categorical statement is logically equivalent to the denial of its contradictory. Explain.

The next relation we define on the square is contrariety. Contraries are such that both may be false but both cannot be true. The relation of contrariety holds between the A and E forms.



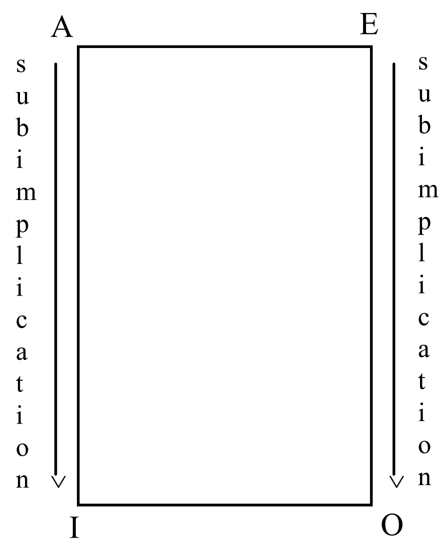
If the A is true, the E must be false. If the E is true, the A must be false. The logical implication holds only from the truth of one form to the falsity of the other. If you only know that one of the contraries is false, you can make no inference to the truth or falsity of the other. When no logical implication holds from one statement to the other, we say the truth value of the second statement is undetermined.

The next relation we examine is subcontrariety. Subcontraries are such that both may be true, but both cannot be false. Subcontrariety holds between the I and O forms.



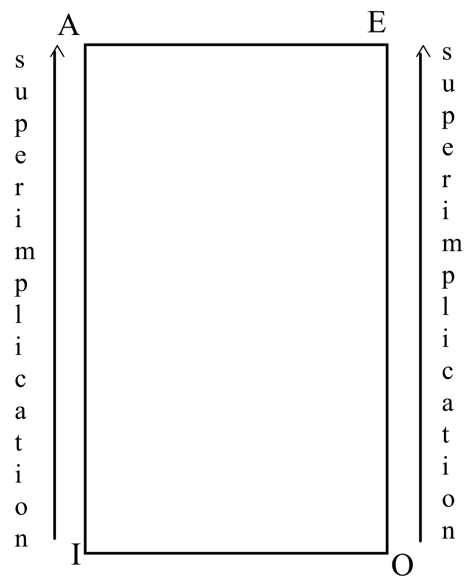
If the I is false, the O must be true. If the O is false the I must be true. No implication holds between the truth of one subcontrary and the truth value of the other.

Subimplication (also called subalternation) holds between the universal and its particular. It is such that if the universal is true, the particular must also be true.



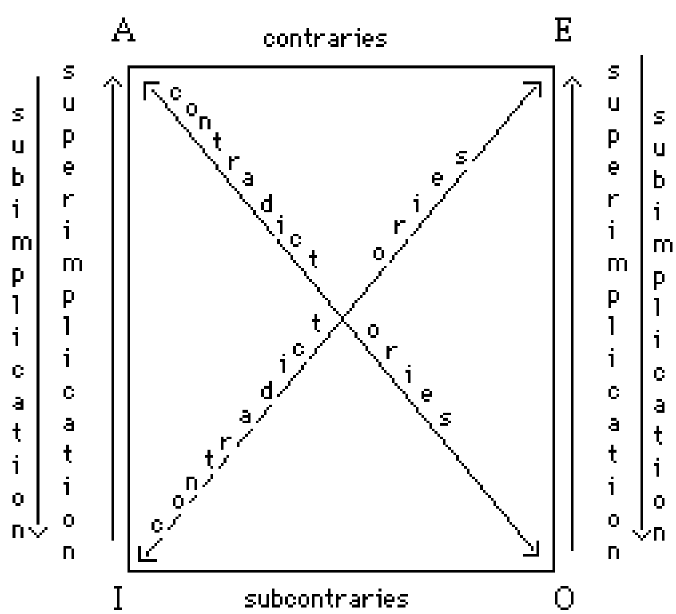
If the A is true, the I must be true. If the E is true, the O must be true. The implication holds only from the truth of the universal to the true of the particular. If you know only that the particular is true, the universal is undetermined. If you know only that the universal is false, the particular is undetermined.

The final relation on the traditional square of opposition is superimplication (or superalternation). The relation here is from the falsity of the particular to the falsity of the universal.



If the I is false, the A must be false. If the O is false, the E must be false. The implication does not hold from the truth of the particular to the truth value of the universal. If the I is true, the A is undetermined. If the O is true, the E is undetermined.

Here is the completed square of opposition.



Exercise 10.6

I. Using the traditional square of opposition, and assuming "All joggers are healthy people" is true, determine whether the following statements are true, false, or undetermined.

1. No joggers are healthy people.
2. Some joggers are not healthy people.
3. Some joggers are healthy people.
4. It is false that some joggers are not healthy people.
5. It is false that no joggers are healthy people.
6. It is false that some joggers are healthy people.

II. Using the traditional square of opposition, and assuming "All joggers are healthy people" is false, determine whether the following statements are true, false, or undetermined.

1. No joggers are healthy people.
2. Some joggers are not healthy people.
3. Some joggers are healthy people.
4. It is false that some joggers are not healthy people.
5. It is false that no joggers are healthy people.
6. It is false that some joggers are healthy people.

III. Using the traditional square of opposition, and assuming "Some joggers are healthy people" is true, determine whether the following statements are true, false, or undetermined.

1. No joggers are healthy people.
2. Some joggers are not healthy people.
3. All joggers are healthy people.
4. It is false that some joggers are not healthy people.
5. It is false that no joggers are healthy people.
6. It is false that all joggers are healthy people.

IV. Using the traditional square of opposition, and assuming "Some joggers are healthy people" is false, determine whether the following statements are true, false, or undetermined.

1. No joggers are healthy people.
2. Some joggers are not healthy people.
3. All joggers are healthy people.
4. It is false that some joggers are not healthy people.
5. It is false that no joggers are healthy people.
6. It is false that all joggers are healthy people.

V. Using the traditional square of opposition, and assuming "Some joggers are not healthy people" is true, determine whether the following statements are true, false, or undetermined.

1. No joggers are healthy people.
2. All joggers are healthy people.
3. Some joggers are healthy people.
4. It is false that all joggers are healthy people.
5. It is false that no joggers are healthy people.
6. It is false that some joggers are healthy people.

VI. Using the traditional square of opposition, and assuming "Some joggers are not healthy people" is false, determine whether the following statements are true, false, or undetermined.

1. No joggers are healthy people.
2. All joggers are healthy people.
3. Some joggers are healthy people.
4. It is false that all joggers are healthy people.
5. It is false that no joggers are healthy people.
6. It is false that some joggers are healthy people.